Noncausal Modeling and Closed-Loop Optimal Input Design
Cross-Directional Processes of a Paper Machine

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Introduction

The paper machine

- Transforms a slurry of pulp fiber into a uniform sheet of paper through a series of dewatering and pressing operations.
- Can be over 100 m long in the machine-direction (MD), producing a sheet over 10 m wide in the cross-direction (CD) at rates exceeding 30 m/s [1].

Figure 1: Schematic of a Fourdrinier paper machine [3]
Introduction

CD actuators

- Spatial variations are controlled by CD actuators distributed across the width of the paper machine.
- Headbox dilution profiling valves and induction heating profilers are two primary CD actuators.

Figure 2: Headbox dilution valves (top) and induction profilers (bottom)
Cross-Directional Process

Steady-state model

\[ y_{ss} = G_{ss}u_{ss} + \Phi e_{ss}, \]  

(1)

where each column of \( G_{ss} \in \mathbb{R}^{m \times m} \) is the sampled impulse response (IR) of a single actuator and all actuators are assumed to have identical and symmetric IR coefficients.

\[ K_{ss} = -Q_3^{-1} \alpha_K G_{ss} Q_1, \]  

(2)

where \( Q_1 \) and \( Q_3 \) are weighting matrices that penalize deviation from set-point and steady-state MV offset, respectively.

Figure 3: Closed-loop control system
Cross-Directional Process

Closed-loop

- MPC ($K_{ss}$) assumed to operate linearly with no active constraints.
- We have the following closed-loop CD process:

$$y_{ss} = (I + G_{ss}K_{ss})^{-1} G_{ss}K_{ss}r + (I + G_{ss}K_{ss})^{-1} v_{ss}, \quad (3)$$
$$u_{ss} = (I + K_{ss}G_{ss})^{-1} r - (I + K_{ss}G_{ss})^{-1} K_{ss}v_{ss}, \quad (4)$$

where $r \in \mathbb{R}^m$ is the spatial excitation signal to be designed.

- **Challenge:** Large input-output dimensions and large number of parameters in $G_{ss}$.

- **Solution:** Use a noncausal scalar transfer function to represent steady-state CD actuator response and reduce complexity.
Causal Scalar Representation

Noncausal Scalar Model

- Scalar noncausal finite IR (FIR) model from any column of $G_{ss}$ represents spatial impulse response of an actuator, i.e.,

$$g(\lambda, \lambda^{-1}) = g_{-n}\lambda^{-n} + \ldots + g_0 + \ldots + g_n\lambda^n,$$

where $n < m$ is a truncated index representing significant coefficients and $g_i = g_{-i}$.

- $n$ is large so a parsimonious noncausal transfer function is used to approximate $g(\lambda, \lambda^{-1})$ as

$$\bar{g}(\lambda, \lambda^{-1}) = \frac{B(\lambda)B(\lambda^{-1})}{A(\lambda)A(\lambda^{-1})},$$

$$B(\lambda^{-1}) = b_0 + b_1\lambda^{-1} + \ldots + b_{n_b}\lambda^{-n_{b}},$$

$$A(\lambda^{-1}) = 1 + a_1\lambda^{-1} + \ldots + a_{n_a}\lambda^{-n_{a}},$$

where $n_a$ and $n_b$ are the orders of $A(\lambda^{-1})$ and $B(\lambda^{-1})$, respectively.
Similarly for \( k(\lambda, \lambda^{-1}) \) we have

\[
\bar{k}(\lambda, \lambda^{-1}) = \frac{F(\lambda)F(\lambda^{-1})}{E(\lambda)E(\lambda^{-1})},
\]

\[F(\lambda^{-1}) = f_0 + f_1 \lambda^{-1} + \ldots + f_{nf} \lambda^{-nf},\]

\[E(\lambda^{-1}) = 1 + e_1 \lambda^{-1} + \ldots + e_{ne} \lambda^{-ne},\]

where \( n_e \) and \( n_f \) are the orders of \( E(\lambda^{-1}) \) and \( F(\lambda^{-1}) \), respectively.

High dimensional MIMO steady-state closed-loop model replaced by scalar noncausal transfer functions, i.e.,

\[
y(x) = \frac{\bar{g}}{1 + \bar{g} k} r(x) + \frac{1}{1 + \bar{g} k} v(x),
\]

\[
u(x) = \frac{1}{1 + \bar{g} k} r(x) - \frac{\bar{k}}{1 + \bar{g} k} v(x),
\]

where \( x \) stands for the spatial coordinate.
Consider the following noncausal Box-Jenkins model:

\[
y(x) = \frac{M(\lambda)M(\lambda^{-1})}{N(\lambda)N(\lambda^{-1})} r(x) + \frac{R(\lambda)R(\lambda^{-1})}{S(\lambda)S(\lambda^{-1})} e(x),
\]

(14)

where \(\{e(x), x = 1, \ldots, m\}\) is a Gaussian white noise sequence.

Assuming all polynomials have no zeros on the unit circle and are minimum phase, there exist causal polynomials \(\tilde{M}_y(\lambda^{-1}), \tilde{N}_y(\lambda^{-1}), \tilde{R}_y(\lambda^{-1}), \tilde{S}_y(\lambda^{-1})\), a white noise sequence \(\{\tilde{e}_y(x)\}\) and a stochastic sequence \(\{\tilde{y}(x)\}\) with the same spectra as \(\{y(x)\}\) such that,

\[
\tilde{y}(x) = \frac{\tilde{M}_y(\lambda^{-1})}{\tilde{N}_y(\lambda^{-1})} r(x) + \frac{\tilde{R}_y(\lambda^{-1})}{\tilde{S}_y(\lambda^{-1})} \tilde{e}_y(x),
\]

(15)

where \(N(\lambda)N(\lambda^{-1})\pi_N = N^2(\lambda^{-1}), \tilde{N}(\lambda^{-1}) = N^2(\lambda^{-1})\) and the same also holds for \(M(\lambda), R(\lambda),\) and \(S(\lambda)\).
Causal Scalar Representation

Causal Equivalent Model

- We have \( \tilde{y}(x) = \frac{\pi_M}{\pi_N} y(x) \), \( \tilde{e}(x) = \frac{\pi_M \pi_S}{\pi_N \pi_R} e(x) \) where \( \pi_N = \prod_i \frac{\lambda^{-1} - \beta_i}{\lambda - \beta_i} \) and \( \pi_M, \pi_R \) and \( \pi_S \) are defined in a similar fashion.

- The input signal \( u(x) \) can also be represented through causal filters, i.e.,

\[
\tilde{u}(x) = \frac{\tilde{M}_u(\lambda^{-1})}{\tilde{N}_u(\lambda^{-1})} r(x) + \frac{\tilde{R}_u(\lambda^{-1})}{\tilde{S}_u(\lambda^{-1})} \tilde{e}_u(x),
\]

(16)

where \( \{ \tilde{u}(x) \} \) and \( \{ u(x) \} \) have the same spectra.
Consider the noncausal model ($\theta$ is the parameter in compact set $\Omega$)

$$y(x) = \bar{g}(\lambda, \lambda^{-1}, \theta)u(x) + \bar{h}(\lambda, \lambda^{-1}, \theta)e(x),$$  \hspace{1cm} (17)

where $e(x)$ is Gaussian white noise and data is generated in closed-loop and all relevant transfer functions are uniformly stable.

Then, as $m \to \infty$ ($m$ is the number of measurement bins),

$$\sup_{\theta \in \Omega} |L^m_{\hat{y}}(\hat{y}) - L^m_y(y)| \xrightarrow{w.p.1} 0,$$  \hspace{1cm} (18)

$$\sup_{\theta \in \Omega} \left\| \frac{dL^m_{\hat{y}}(\hat{y})}{d\theta} - \frac{dL^m_y(y)}{d\theta} \right\| \xrightarrow{w.p.1} 0,$$  \hspace{1cm} (19)

where $L^m_y(y)$ is the noncausal log-likelihood function and $L^m_{\hat{y}}(\hat{y})$ is the causal-equivalent log-likelihood function [4].

Therefore, the parameter covariances coincide and we may perform optimal input design based on the causal-equivalent model.
Optimal Input Design

- Split \( \theta \) as \( \theta = [\rho^T \eta^T]^T \) and focus on process model parameters \( (\rho) \).

- **Objective:** minimize a function of the parameter covariance of \( \rho \), \( P_\rho \), subject to input and output power constraints, i.e.,

\[
\begin{align*}
\min_{\Phi_r(\omega)} & \quad f_0(P_\rho(\Phi_r(\omega))) \\
\text{s.t.} & \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_u(\omega) d\omega \leq c_u, \\
& \quad \frac{1}{2\pi} \int_{-\pi}^{\pi} \Phi_y(\omega) d\omega \leq c_y,
\end{align*}
\]

(20)

(21)

(22)

where \( c_u \) and \( c_y \) are the power limits on input and output signals.

- Finite dimensional parameterization of \( \Phi_r \), i.e.,

\[ \Phi_r(\omega) = \sum_{k=-m_c}^{m_c} c_k e^{-j\omega k} \geq 0, \quad \forall \omega, \]

(23)

where \( c_k, k = -m_c, \ldots, m_c \), are the parameters, and \( m_c \) is the selected number of parameters [5].

- Choosing \( f_0(\cdot) \) to be convex the resulting optimization is convex.
Spatial actuator response is nonlinear with four parameters, i.e., gain ($\gamma$), width ($\xi$), divergence ($\beta$), attenuation ($\alpha$) [2].

Comparing three methods

1. **Optimal input design:** causal-equivalent model, excitation amplitude constrained to $\leq \pm 10$.
2. **Bump excitation:** amplitudes alternate between $+10$ and $-10$.
3. **White noise:** designed with the same variance as the optimal input.

For computational efficiency model orders are specified as $n_b = n_f = 1$ and $n_a = n_e = 2$.

Process model is identified in 100 Monte-Carlo simulations.
Case Study

- High order models can improve accuracy with a computation cost.

**Figure 4:** IR of a single actuator (red) and noncausal estimate (blue).
Case Study

- Large spectrum amplitude in the cross-over frequency enables better excitation.

Figure 5: Optimal input spectrum from causal-equivalent model
Case Study

**Impulse Responses of the Process Under Optimal Input**

![Graph showing impulse responses under optimal input]

**Impulse Responses of the Process Under Bumped Input**

![Graph showing impulse responses under bumped input]

**Impulse Responses of the Process Under White Noise Input**

![Graph showing impulse responses under white noise input]
Summary

- Averaged errors ($\bar{\epsilon}$)
  1. Optimal input design: $\bar{\epsilon} = 0.0643$
  2. Bump excitation: $\bar{\epsilon} = 1.3344$
  3. White noise: $\bar{\epsilon} = 0.4479$

- **Noncausal model**: circumvents large dimension of MIMO CD process.

- **Causal-equivalent modeling**: facilitates traditional optimal input design methods.


